SIMILARITY OF NON-NEWTONIAN FLOWS. IV.*
PSEUDOSIMILARITY AND GENERALIZED TRANSPORT CORRELATIONS

O.WEIN, K.WICHTERLE, P.MITSCHKA and J.ULBRECHT**

Institute of Chemical Process Fundamentals,
Czechoslovak Academy of Sciences, Prague - Suchdol

Received October 26th, 1970

The principle of pseudosimilarity of creeping flows of Generalized Newtonian Fluids is formulated here on the basis of formal properties of the power-law model and the Metzner–Rabinowitsch flow models which postulates an approximate hydrodynamic similarity of these flows with the only condition of identity of the apparent flow index n^* regardless the actual shape of the viscosity characteristics. Possibilities of an analogous approach are also discussed for the case when inertia forces or time-dependent rheological phenomena cannot be neglected.

The final step of an engineering solution of hydrodynamic and transport problems is usually the expression of dependences in the form of dimensionless correlations in such a way as to be valid for the widest possible range of parameters characterizing the considered situation and for the widest possible variety of materials coming into consideration.

Such correlations in problems concerned with the flow of non-Newtonian liquids are rather difficult. A unified correlation for a group of non-Newtonian liquids is obviously subject to hydrodynamic similarity of corresponding flow situations and thus to rheological similarity[1] of all considered liquids. While for Newtonian liquids the condition of rheological similarity is automatically fulfilled[2], for non-Newtonian liquids it is highly improbable that even two materials could be found with exactly similar viscosity characteristics[2] without considering a great variety of other rheological phenomena which sometimes affect the flow course and which must be taken into consideration when modelling.

Since the possibility of scaling-up (and thus of unified correlating) of non-Newtonian flows in an exact way is *de facto* impossible, engineers try to present methods of approximate modelling which are based on the description of rheological properties covering basical rheological phenomena for the considered group of liquids but which enable to form inside such a group sufficiently wide subgroups of materials,

* Part III: This Journal *37*, 1467 (1972).

** Present address: University of Salford, Salford, U.K.

mutually "approximately rheologically similar" in a suitably chosen sense. With this aim there were suggested already innumerable number of so-called "non-Newtonian models" *e.g.* two-to-four-parameter formulae for description of the course of viscosity characteristics (see for instance reviews in monographies[3,4]) as well as simple models describing at least qualitatively more complex phenomena of thixotropy[5] and viscoelasticity[6–9].

We admit that it is useful to use three- and more parameter formulae in evaluating data obtained by rheometrical experiments. However, we do not consider desirable to introduce these formulae into engineering correlations. Results of a number of problems concerning non-Newtonian flow and heat transfer (see for example the works of Metzner and co-workers[10–16]) show that for these purposes the power law model can be used with sufficient accuracy when its parameters *i.e.* the consistency coefficient K and flow index n are suitably chosen.

This work, which is closely related to our previous papers[2,17] tries to formulate explicitly the reasons leading just to the choice of the power-law model as a tool for generalized engineering correlations on non-Newtonian flow and heat transfer.

Kinematic Pseudosimilarity

It is known[3,4,10] that for flow through a pipe and a slit the so-called Metzner–Rabinowitsch relations can be derived, according to which the value of velocity gradient on the wall can be determined if the dependence of the pressure drop on volumetric flow rate is known, for the given liquid and the given flow arrangement.

We have given in our previous work[17] other examples of these hydrodynamic situations which are summarily called MR-flows and we have shown that it is possible, without regard to a detailed course of the viscosity characteristics, to determine the normalized velocity gradient on the wall as a unique function of an apparent flow index

$$n^* = \frac{\mathrm{d} \ln P_c}{\mathrm{d} \ln U_c/R_c}, \qquad (I)$$

where P_c, U_c and R_c are macroscopic parameters of a given hydrodynamic situation[1,17].

Agreement in values of normalized velocity gradients at the wall together with normalizing conditions causes that even the courses of normalized velocity profiles for substantially different type of viscosity characteristics do not differ mutually by more than 20%, while the agreement in their courses close to the walls is considerably better. Also in other hydrodynamic situations like in the Couette flow[18,19], in axial flow through annulus[19,20], and others[19,21], where normalized gradients on the walls are not unique functions of the apparent flow index and which exhibit according to the character of the viscosity characteristics a certain scatter around the value resulting from the power-law model, relations of the following type are quite satisfactory, *i.e.* within the limits of engineering accuracy * ± 10%

* By deviations ±10% are understood deviations from the curve for the power-law flow model due to reasons explained later.

$$w \approx w_a\left(\xi, \frac{\mathrm{d}\ln P_c}{\mathrm{d}\ln U_c/R_c}\right). \tag{2}$$

We have shown in our previous work[2] that the result of solution of the mathematical model of the creeping flow of the power-law liquid in the form

$$\frac{P_c}{K(U_c/R_c)^n} = \mathrm{Cr}'(n), \tag{3}$$

$$\boldsymbol{v}^* = \boldsymbol{v}_a^*(\boldsymbol{r}^*, n), \tag{4}$$

cannot be understood only as a result obtained for one arbitrarily chosen two-parameter model, but as the most general type of solution for the class of automorphic visco-inelastic models. So if the courses of normalized velocity fields (2) are compared with the power-law courses (*i.e.* courses determined for the power-law model) this is not done only by chance but with an effort to introduce into generalized correlations as much as possible formal properties of automorphic models which are so useful for engineering considerations.

Automorphy of the power-law model is manifested for creeping flow, by the same — with exception for numerical factor $\mathrm{Cr}'(n)$ — relation between operational parameters

$$P_c = \mathrm{Cr}'(n)\, K(U_c/R_c)^n \tag{5}$$

and the viscosity characteristics

$$\tau = KD^n. \tag{6}$$

Moreover the apparent flow index n^*, for which according to (5) for creeping flows the relation

$$n^* = \frac{\mathrm{d}\ln P_c}{\mathrm{d}\ln U_c/R_c} = n, \tag{7}$$

is valid, is the single parameter of the velocity field

$$\boldsymbol{v}^* = \boldsymbol{v}_a^*\left(\boldsymbol{r}^*, \frac{\mathrm{d}\ln P_c}{\mathrm{d}\ln U_c/R_c}\right). \tag{8}$$

So far, we have shown that relation (2) is valid approximately for a certain class of one-dimensional flow situations and for an arbitrarily chosen type of viscosity characteristics; relations of the same form (8) are then valid for a certain class of viscosity characteristics and for any type of the creeping flow.

This leads us to a formulation of a hypothesis of *kinematic pseudosimilarity*[20]: "If between two creeping flows of Generalized Newtonian Fluids in a kinematically similar hydrodynamic arrangements pseudosimilarity is preserved, *i.e.*

$$n^* = \mathrm{idem}, \tag{9}$$

then their normalized velocity fields are equivalent" in the limits of engineering accuracy. As a result of this hypothesis it is possible to approximate the normalized

velocity fields for a known n^* for the given hydrodynamic arrangement by the power-law velocity field, *i.e.* in the form (*8*).

Dynamic and Rheological Pseudosimilarity

For power-law creeping flows also the macroscopic momentum balance has several peculiarities when compared with the general shape of viscosity characteristics. The macroscopic quantity P_c can be introduced according to the type of hydrodynamic situation[1,17] as a pressure drop along a characteristic length of the channel, as the overall acting force related to the unit of the wall surface, or as a moment of force related to the volume unit of body rotating in the liquid, eventually as multiples of these quantities. From macroscopic momentum balances[3] it follows that this quantity having the above mentioned physical meaning can be expressed in the form

$$P_c = \chi (1/\Omega) \iint_{\Omega} \mathbf{e}(\boldsymbol{r}) \,.\, (-P(\boldsymbol{r})\, \boldsymbol{E} + \tau) \,.\, \mathrm{d}\boldsymbol{\Omega} \,. \tag{10}$$

The integral momentum balance in dimensionless form can be thus written as

$$\mathrm{Cr}'(n) = \frac{\chi \iint_{\Omega} \mathbf{e}(\boldsymbol{r}^*) \,.\, \tau^*(\boldsymbol{r}^*, n) \,.\, \mathrm{d}\boldsymbol{\Omega}^*}{1 + \chi \iint_{\Omega} \mathbf{e}(\boldsymbol{r}^*) \,.\, (P^*(\boldsymbol{r}^*, n)\, \boldsymbol{E}) \,.\, \mathrm{d}\boldsymbol{\Omega}^*} \,. \tag{11}$$

According to Eq. (*10*) P_c has been introduced as a certain mean value of shear stress on the wall, resp. as its multiple. If we denote the characteristic mean value of shear stress on the wall as τ_w, then

$$\tau_w = \nu P_c \,, \tag{12}$$

where ν is a geometrical simplex. If we define the characteristic velocity gradient on the wall D_w so that the relation

$$\tau_w = K\, D_w^n \,, \tag{13}$$

holds then it can be shown by substitution from relations (*13*) and (*12*) into Eq. (*11*) that for creeping power-law flows generally

$$D_w = \phi(n)\, U_c / R_c \,. \tag{14}$$

holds.

Relations (*12*) and (*14*) are most important for our considerations. Analogically as in the case of kinematic similarity considered in the previous chapter, also here

analogous relations like (*12*) and (*14*) for the power-law flows are valid as well as for MR-flows of liquids with an arbitrary of course viscosity characteristics. Between parameters P_c and τ_w for MR-flows[17] the relation

$$\tau_w = \nu P_c \,, \tag{15}$$

holds, where $\nu = 1$ and the relation between D_w and U_c/R_c is expressed for them by Metzner–Rabinowitsch relations[17]

$$D_w = \phi(n^*)\, U_c/R_c \,. \tag{16}$$

Relations of the type (*15*) and (*16*) have been found to be valid for a number of one-dimensional flow situations[19,20,21] regardless of the form of viscosity characteristics, with an insignificant scatter around the value $\phi(n)$ for power-law models of corresponding flow situations, and with the difference that in relation (*15*) is not $\nu = 1$ but that it has a meaning of a geometrical simplex.

The mentioned results lead us to formulation of the hypothesis of *dynamic pseudosimilarity*:

"Ratios of operational parameters P_c resp. U_c/R_c and of their rheological counterparts τ_w resp. D_w are for pseudosimilar flows approximately constant, *i.e.* the following relations may be written

$$\tau_w/P_c \approx \nu \,, \tag{17}$$

$$D_w/(U_c/R_c) \approx \phi(n^*) \,, \tag{18}$$

while ν is a mere geometrical simplex".

For the power-law liquid the rheological constant n,

$$n = \frac{\mathrm{d} \ln \tau}{\mathrm{d} \ln D} \,, \tag{19}$$

is equal to n^*. However, this is not valid for other viscosity characteristics. Let us introduce another parameter n_{rh} as the logarithmic derivative of the viscosity characteristics, by relation

$$n_{rh} \equiv \frac{\mathrm{d} \ln \tau[D]}{\mathrm{d} \ln D} = n_{rh}[\tau] = n_{rh}[D] \,. \tag{20a}$$

By substituting (*20a*) for $\tau_w = \nu \,.\, P_c$, $D_w = \phi(n^*).\, U_c/R_c$ into relations (*17*) and (*18*) we get

$$\frac{1}{n_{rh}} = \frac{d \ln D_w}{d \ln \tau_w} = \frac{d \ln (\phi(n^*) U_c/R_c)}{d \ln P_c} . \qquad (21)$$

The equality

$$n_{rh} = n^* \qquad (22)$$

thus exactly holds only when the second derivative in relation between $\ln U_c/R_c$ and $\ln P_c$ vanishes (*i.e.* in inflex points or in straight-line segments), $d \ln \phi(n^*)/d \ln P_c = 0$. But approximately the equality (*22*) of rheological pseudosimilarity is valid, which we formulate as the last of postulates:

"Viscosity characteristics of liquids the flows of which are pseudosimilar for certain U_c/R_c resp. P_c, have in the point $\tau_w = \nu P_c$, $D_w = \phi(n^*) U_c/R_c$ the same logarithmic derivatives":

$$\left.\frac{d \ln \tau}{d \ln D}\right|_{D_w, \tau_w} = n_{rh} \approx n^* . \qquad (23)$$

Since, as follows from the results already presented, the choice of parameter P_c resp. τ_w for other than MR-flows is arbitrary the question of dynamic and rheological pseudosimilarity can be related and the reversed question can be put: "How to define suitably P_c or τ_w for the considered hydrodynamic situation to fulfil at best the conditions of dynamic and rheological pseudosimilarity for an arbitrary viscosity characteristics?"

This question cannot be of course solved in general but it is necessary to consider it separately for each type of flow situation with regard to the structure of macroscopic momentum balances (*10*) and (*11*).

Correlations Based on Pseudosimilarity

The *kinematic pseudosimilarity* is the starting point for construction of generalized transport correlations where the only parameter characterizing the effect of rheological properties of the flowing liquid on the convective heat and mass transfer is the apparent flow index n^*. This is subject to the fact that equations of heat and mass transfer (in the cases when the hydrodynamics of flow is not to a great extent reversely affected by heat and mass transfer) can be normalized by use of parameters U_c/R_c and of a characteristic values of the potential of the respective driving forces for transfer. The final dimensionless mathematical model of transfer includes the normalized velocity field with the apparent flow index n^* as the determining parameter according to the assumptions of pseudosimilarity and no other parameters characterizing the rheological properties. In formulating transport correlations which usually represent macroscopic heat and mass transfer balances in dimensionless form, the above mentioned step can be neglected and in generalized correlations, where the

dependent variable is the dimensionless transport coefficient (for example the Nusselt number) only the apparent flow index n^* can be used.

Dynamic and rheological pseudosimilarity enable by formulating the generalized correlations the determination of the function $\phi(n^*)$ and ν from experimental data on flow dynamics and the values of the apparent flow index n^* which is the only parameter in generalized correlations including rheological properties.

In the reversed procedure – by using the mentioned transport correlations – the dynamic pseudosimilarity enables to determine, on the basis of the known correlation relations and the known viscosity characteristics, the value of parameter n^*, to estimate the dynamic behaviour of the proposed system (*i.e.* the relation between P_c and U_c/R_c) and of course to solve the considered transport problem.

According to definition (*20a*) the viscosity characteristics can be transformed into the form

$$n_{rh} = n_{rh}[D] . \tag{20b}$$

According to the postulates of kinematic pseudosimilarity

$$D_w = \phi(n^*) \left(\frac{U_c}{R_c}\right)^n \tag{18b}$$

holds so that for the known function $\phi(n^*)$ $n^* = n_{rh}$ and $D = D_w$ can be determined by simultaneous solution of Eqs (*18b*) and (*20b*). From rheograms the value $\tau_w = \tau[D_w]$ can be taken and in a subsequent step also the value of parameter P_c according to Eq. (*17*) can be determined.

From the presented procedure it follows that for the use of generalized transport correlations it is necessary to know the course of the function $\phi(n^*)$ and of the factor ν, so that their determination is an inseparable part of the constructing of all generalized correlations.

Already the formulation of the characteristic stress τ_w in Eq. (*17*) resp. the formulation of parameter ν includes implicitly the assumption that with the use of the rheogram for the value P_c of each liquid a value τ_w can be found so that the postulates of pseudosimilarity is satisfied. This assumption of pseudosimilarity must be verified by testing pseudosimilarity for each hydrodynamic arrangement separately.

In cases when the solution of the mathematical model for the power-law flow is known, the course of function $\phi(n^*)$ and of ν can be determined numerically and usually also testing is possible, *i.e.* the comparison of an approximate solution on the basis of pseudosimilarity with the exact solution for several suitably chosen viscosity characteristics. In cases when such solution is not available, two procedures are at hand: *1.* It can be assumed that $\nu = 1$ (this is generally satisfied *e.g.* in the flow through channels). For the measured values of the variables (P_c, U_c/R_c) for given liquids n^* can be determined either by derivation of these data or on basis of Eq. (*17*), (*23*). By comparing both values the assumption $\nu = 1$ can be verified, if several liquids of different courses of viscosity characteristics have been used in the measurement. The course of function $\phi(n^*)$ is determined for the known viscosity characteristics of the used liquids according

to Eq. (*18a*). 2. By experimenting with a number of liquids with widely different courses of viscosity characteristics the values ν can be also determined. Viscosity characteristics of these liquids can be transformed into pairs of functions

$$n_{\mathrm{rh}} = n_{\mathrm{rh}}[\tau] , \tag{24}$$

$$K_{\mathrm{rh}} = \frac{\tau}{(D[\tau])^{n_{\mathrm{rh}}[\tau]}} = K_{\mathrm{rh}}[\tau] . \tag{25}$$

On the basis of Eqs (*17*), (*23*), (*24*) the value ν can be determined for each experimentally determined triple of values (P_c, U_c/R_c, n^*) as a solution of equation (*26*):

$$n_{\mathrm{rh}}[P_c \nu] - n^* = 0 , \tag{26}$$

if, of course, the change of n^* in function (*24*) is sufficiently expressive. The value $\phi(n^*)$ can be determined on the basis of relations (*17*), (*18a*), (*25*) from equation (*27*)

$$\phi(n^*) = \frac{R_c}{U_c} \left(\frac{P_c}{K_{\mathrm{rh}}[P_c \nu]} \right)^{1/n_{\mathrm{rh}}[P_c \nu]} , \tag{27}$$

where ν is already considered to be known. The scatter of values ν for liquids with different courses of viscosity characteristics gives as well as in the first case information upon the applicability of the conception of pseudosimilarity for the given geometrical arrangement.

TESTING OF PSEUDOSIMILARITY

It was already stressed that relations by which the pseudosimilarity is postulated are exactly valid only for power-law liquids and their validity in general must be tested. If the pseudosimilarity correlations are based on experimenting with rheologically different materials, the deviations from basical assumptions will be directly given by the scatter in the obtained correlations.

In cases when the respective hydrodynamic and transport problems can be solved analytically for a power-law fluid, the limits of applicability of the pseudosimilarity concept can be tested numerically. Since for power-law models the pseudosimilarity is valid exactly, it is useful to test this concept by comparing results of solution for the power-law model with those for other supposed courses of viscosity characteristics. In our previous work[17] we have shown that for the mentioned testing of MR-flows, it is the most suitable to use the Bingham and Eyring model. for this purpose. This is also demonstrated by a several plots in the previous work[17] where *de facto* the kinematic pseudosimilarity for three MR-flows is tested: the Poiseuille flow, translation, and rotation of a cylinder in liquid. Dynamic pseudosimilarity for MR-flows is fulfilled exactly so that for the MR-flows discussed in the paper[17] it remains to test only the rheological pseudosimilarity. Results of this test are obvious from Figs 1—3.

GENERAL ASPECTS OF AUTOMORPHIC NON-NEWTONIAN FLOW MODELS

In the preceding study of possibilities how to construct generalized correlations including the smallest number of rheological parameters as possible, we limited ourselves to the case of creeping flow of non-Newtonian fluids whose viscosity characteristics fully express their rheological properties. But it is obvious that is not the most interesting case of the engineering practice. In cur-

rently used flow arrangements it is not usually possible to neglect the effect of inertia forces acting in the liquid and/or the effect of rheological phenomena dependent on time, like visco-elasticity and thixotropy. Before we try to formulate the principles of generalized correlations also for these more complicated cases, it is advisable to summarize the procedure for the mentioned simpler case.

Basically, a very simple two-parameter rheological model was chosen as a substitute model, and for this the solution of the hydrodynamic or transport problem was found (or this was assumed) including also parameters of the substitute model (in the case of creeping flow the solution includes only a single parameter of the substitute model). By the generalized correlations either the procedure how to calculate parameters of the substitute model on the basis of known

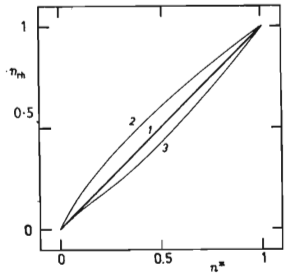

FIG. 1

Dependence of n_{rh} on n^* for Pipe Flow
1 Power-law model, *2* Bingham model, *3* Eyring model.

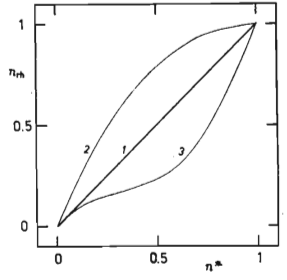

FIG. 2

Dependence of n_{rh} on n^* for Flow in Vicinity of a Cylinder in Axial Motion
1 Power-law model, *2* Bingham model, *3* Eyring model.

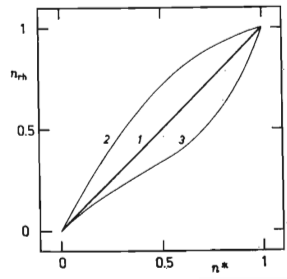

FIG. 3

Dependence of n_{rh} on n^* for Flow in Vicinity of Rotating Cylinder
1 Power-law model, *2* Bingham model, *3* Eyring model.

rheological constitutive relations of specific materials should be determined, or a simple typical experiment for determination of these parameters empirically should be suggested. The indispensable condition for success is to choose a substitute model as simple as possible which qualitatively describe the determining rheological phenomena, and which lead to a description of reality as close as possible.

In our two previous works[2,17] we have shown that for description of visco-inelastic flows the power-law model is objectively the most suitable substitute model just for its automorphic properties. Further on we present how it would be possible in postulating of generalized correlations in more complex case to make use of automorphic substitute models of a more general type.

The automorphic rheological models properties of which were considered in detail in our previous work[2] can be in a most general form described in the form of a functional

$$\overset{\infty}{\underset{t=0}{H_0}}[\tau, \ldots, \tau^{(i)}; \mathbf{D}, \ldots \mathbf{D}^{(i)}; t; n; \alpha_1, \ldots \alpha_N, K] = 0 , \tag{28}$$

which includes only one dimensional material constant, K.
In this case the time differential operators whose symbols are denoted by the superscript in brackets, must have the form

$$\mathbf{X}^{(i)} = \frac{1}{D_i} \frac{\mathrm{D}\mathbf{X}^{(i-1)}}{\mathrm{D}t} ,$$

where D_i are some invariants of kinematic tensors, *i.e.* in automorphic models differential time operations do not introduce into the constitutive relations any parameters with the dimension of time. From the formal point of view, the main advantage of automorphic models is that they enable to write the normalized flow model in the form

$$\mathrm{Re}'\left(\frac{1}{\mathrm{St}} \frac{\partial \mathbf{v}^*}{\partial t^*} + \mathbf{v}^* \nabla^* \mathbf{v}^*\right) = -\mathrm{Cr}' \nabla^* \mathrm{P}^* + \nabla^* \cdot \tau^* , \tag{29}$$

$$\overset{\infty}{\underset{t^*=0}{H_a^*}}[\tau^*, \ldots \tau^{*(i)}; \mathbf{D}^*, \ldots, \mathbf{D}^{*(i)}; t^*; \mathrm{St}; n; \alpha_1, \ldots \alpha_N] = 0 , \tag{30}$$

where

$$t^* = tD_i , \tag{31a}$$

$$\mathbf{D}^* = \mathbf{D}R_c/U_c , \tag{31b}$$

$$\tau^* = \tau/(K(U_c/R_c)^n) , \tag{31c}$$

$$\mathrm{Re}' = \varrho(U_c^{2-n} R_c^n)/K , \tag{32}$$

$$\mathrm{Cr}' = P_c/(K(U_c/R_c)^n) , \tag{33}$$

and where, in comparison with the general case presented in detail in our previous work[1], criteria of dynamic similarity De, Wi, B are not included as independent variable parameters of the dimensionless flow model, because automorphic rheological model does not include constant having the dimension of time.

The mathematical flow model in which the automorphic model (*28*) resp. (*30*) appears as a constitutive relation has the following properties: *1.* It includes only one independent criterion of dynamic similarity in which simultaneously characteristic parameters of the flow situation and material parameters are included. $\mathrm{Re}' \equiv$ idem is thus the only modelling condition, and for modelling

of automorphic flows generally all that has already been said about modelling of visco-inelastic power-law flows is valid. 2. When the effect of inertia forces is neglected the macroscopic balance can be written in the form

$$\mathrm{Cr}' = \mathrm{Cr}'(n, \alpha_1, \ldots \alpha_N) . \tag{34}$$

It is thus possible to determine the flow index n as

$$n = \frac{\mathrm{d} \ln P_c}{\mathrm{d} \ln U_c/R_c} \tag{35}$$

also for time dependent automorphic materials. In this case

$$\boldsymbol{v}^* = \boldsymbol{v}^*(\boldsymbol{r}^*; n; \alpha_1, \ldots, \alpha_N) . \tag{36}$$

is valid.

CONCLUSION

If the substitute rheological model includes parameters K, n, α, the principles of pseudosimilarity can be extended so that instead of the single condition $n^* = \text{idem}$, the system of conditions will be

$$\mathrm{Re}^* \equiv \text{idem} , \tag{37a}$$

$$n^* \equiv \text{idem} , \tag{37b}$$

$$\alpha^* \equiv \text{idem} , \tag{37c}$$

where n^* is determinated as the logarithmic slope of dependence between P_c and U_c/R_c for the creeping flow regime in the same arrangement and with the same value U_c/R_c, or can be read off as $n^* = n_{\mathrm{rh}}[\tau]$ from the viscosity characteristics of the given material for suitably chosen τ_w. Re* is defined by relation (32) where $n = n^*$.

The choice of parameter α^*, which characterizes the time variable properties of the material[17] and to which the question of rheological pseudosimilarity is related, must be solved for each group of materials separately with a common substitute rheological model. For its determination independent rheometric experiments must be made unlike for the apparent flow index n^* which can be in principle evaluated from the results of systematic rheodynamic experiments in a given flow situation.

Individual aspects of this work as well as its overall conception have been much affected by discussions with Prof. H. Steidl, Dr J. Nebřenský, Dr V. Kolář, and Dr J. Šesták.

LIST OF SYMBOLS

$\boldsymbol{D}^*$	normalized shear rate tensor, Eq. (31*b*)
D	scalar shear rate (s^{-1})

D_i suitable invariants of kinematic tensors, having a dimension of time, for different $i = 1, 2$ may, but need not be chosen differently (s^{-1})
D_t one of invariants D_i or their combination chosen to normalize time variables (s^{-1})
D_w characteristic shear rate at the wall (s^{-1})
$\mathbf{e}$ unit vector of variable direction, chosen according to the type of problem
$\mathbf{E}$ unit tensor of second order
H_a automorphic constitutive functional[1,17]
H_a^* normalized functional H_a, Eq. (*30*)
K consistency coefficient $(\text{dyn cm}^{-2} \text{ s}^n)$
K_{rh} apparent consistency coefficient, Eq. (*25*) $(\text{dyn cm}^{-2} \text{ s}^n)$
n flow index (Eq. (*7*)), parameter of power-law model (Eq. (*6*))
n^* apparent flow index according to Eq. (*1*)
n_{rh} apparent flow index determined on basis of rheological pseudosimilarity, Eq. (*20*)
P hydrodynamic potential (dyn cm^{-2})
$P^* = P/P_c$ hydrodynamic potential (normalized)
P_c characteristic dynamic parameter[1] (dyn cm^{-2})
R_c characteristic length[1] (cm)
$\mathbf{r}$ radiusvector (cm)
$\mathbf{r}^*$ normalized radiusvector
t time (s)
t^* normalized time variable, Eq. (*31a*)
U_c characteristic velocity (cm s^{-1})
w dimensionless velocity in one-dimensional flow situations
$w_a(\xi)$ course of normalized velocity w according to the power-law model
$\mathbf{v}$ velocity (cm s^{-1})
$\mathbf{v}^* = \mathbf{v}/U_c$ normalized velocity
$\mathbf{v}_a^*(\mathbf{r}^*)$ course of velocity field normalized according to the power-law model
α, α_N parameters of automorphic rheological models
χ dimensionless parameter in formulation of macroscopic momentum balance
ξ dimensionless coordinate for one-dimensional flows
ν dimensionless parameter with value dependent only on the kinematic flow arrangement, Eq. (*12*)
$\phi(n)$ function which is the solution of the power-law flow model; characteristic function in the Metzner–Rabinowitsch equations[17]
Ω cross-sectional flow area
τ shear stress tensor
τ second invariant of shear stress tensor
τ_w characteristic mean stress on the wall
τ^* normalized shear stress tensor for automorphic rheological models
∇ nabla operator, Eq. (*31c*)
$\nabla^* = R_c \nabla$ normalized nabla operator

REFERENCES

1. Wein O., Wichterle K., Nebřenský J., Ulbrecht J.: This Journal *37*, 784 (1972).
2. Wein O., Mitschka P., Ulbrecht J.: This Journal *37*, 1106 (1972).
3. Bird R. B., Stewart W. E., Lighfoot E. N.: *Transport Phenomena*. Wiley, New York 1960.
4. Skelland A. H. P.: *Non-Newtonian Flow and Heat Transfer*. Wiley, New York 1967.
5. Fredrickson A. G.: A.I.CH.E. J. *16*, 436 (1970).

6. Metzner A. B., White J. L., Denn M. M.: A.I.CH.E. J. *12*, 863 (1966).
7. Bird R. B., Carreau P. J.: Chem. Eng. Sci. *23*, 427 (1968).
8. Carreau P. J., Macdonald I. F., Bird R. B.: Chem. Eng. Sci. *23*, 981 (1968).
9. Bogue D. C., Doughty J. D.: Ind. Eng. Chem. Fundam. *5*, 243 (1966).
10. Metzner A. B., Reed J. C.: A.I.CH.E. J. *1*, 434 (1955).
11. Metzn11. Metzner A. B., Vanghan R., Houghton G. L.: A.I.CH.E. J. *3*, 92 (1957).
12. Metzner A. B., Otto R. E.: A.I.CH.E. J. *3*, 3 (1957).
13. Metzner A. B.: Ind. Eng. Chem. *49*, 1429 (1957).
14. Metzner A. B., White J. L.: A.I.CH.E. J. *11*, 989 (1965).
15. Marshall R. J., Metzner A. B.: Ind. Eng. Chem. Fund. *6*, 393 (1967).
16. Metzner A. B., Uebler E. A., Fong C. F. C. M.: A.I.CH.E. J. *15*, 750 (1969).
17. Wein O., Mitschka P., Ulbrecht J.: This Journal *37*, 1471 (1972).
18. Krieger I. M., Maron S. H.: J. Appl. Phys. *25*, 72 (1954).
19. Kozicki W., Pasari S. N., Rao A. R. K., Tiu C.: Chem. Eng. Sci. *25*, 41 (1969).
20. Wein O., Nebřenský J., Wichterle K., Ulbrecht J.: *Paper Presented at the 3rd International CHISA Congress, Mariánské Lázně, September 1969.*
21. Lescarboura J. A., Eichstadt F. J., Swift G. W.: A.I.CH.E. J. *13*, 169 (1967).

Translated by M. Rylek.